

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad A^{-1} = \begin{bmatrix} A^{11} & A^{12} \\ A^{21} & A^{22} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} = I$$

$$(1) A_{11}A^{11} + A_{12}A^{21} = I$$

$$(2) A_{11}A^{12} + A_{12}A^{22} = 0 \rightarrow A_{11}A^{12} = -A_{12}A^{22} \quad (3)$$

$$(3) A_{21}A^{11} + A_{22}A^{21} = 0 \rightarrow A_{22}A^{21} = -A_{21}A^{11} \quad (4)$$

$$(4) A_{21}A^{12} + A_{22}A^{22} = I$$

طرفین (3) را در A_{11}^{-1} ضرب کنید $A_{11}^{-1}A_{11}A^{12} = -A_{11}^{-1}A_{12}A^{22} \Rightarrow A^{12} = -A_{11}^{-1}A_{12}A^{22} \quad (5)$

طرفین (4) را در A_{22}^{-1} ضرب کنید $A_{22}^{-1}A_{22}A^{21} = -A_{22}^{-1}A_{21}A^{11} \Rightarrow A^{21} = -A_{22}^{-1}A_{21}A^{11} \quad (6)$

با جایگزینی (5) در (1) :

$$A_{11}A^{11} + A_{12}(-A_{22}^{-1}A_{21}A^{11}) = I$$

$$\Rightarrow (A_{11} - A_{12}A_{22}^{-1}A_{21})A^{11} = I$$

$$\Rightarrow A^{11} = (A_{11} - A_{12}A_{22}^{-1}A_{21})^{-1}$$

با جایگزینی (6) در (2)

$$A_{21}(-A_{11}^{-1}A_{12}A^{22}) + A_{22}A^{22} = I$$

$$(-A_{21}A_{11}^{-1}A_{12} + A_{22})A^{22} = I$$

$$\Rightarrow A^{22} = (-A_{21}A_{11}^{-1}A_{12} + A_{22})^{-1}$$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$$

مثال ۱. وارون ماتریس A را محاسبه کنید

$$A^{11} = (A_{11} - A_{12} A_{22}^{-1} A_{21})^{-1}$$

$$A = \begin{bmatrix} \boxed{1} & \boxed{0} & \boxed{2} \\ \boxed{-1} & \boxed{1} & \boxed{0} \\ \boxed{2} & \boxed{1} & \boxed{2} \end{bmatrix}$$

A_{11} A_{12}
 A_{21} A_{22}

$A_{11}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\left[\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 2 & 1 \end{pmatrix} \right]^{-1}$$

$$= \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}^{-1} = \frac{-1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} -1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$$

$$A^{22} = (-A_{21} A_{11}^{-1} A_{12} + A_{22})^{-1}$$

$$= (- (2 \ 1) \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 2)^{-1} = (-4)^{-1} = -1/4$$

$$A^{21} = -A_{22}^{-1} A_{21} A^{11} = -\frac{1}{2} (2 \ 1) \begin{pmatrix} -1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$$

$$= \begin{pmatrix} 3/4 & 1/4 \end{pmatrix}$$

$$A^{12} = -A_{11}^{-1} A_{12} A^{22} = -\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} (-1/4)$$

$$= -\begin{pmatrix} 2 \\ -2 \end{pmatrix} (-1/4) = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 \\ 3/4 & 1/4 & -1/4 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$$

و جواب