

①

$$\mu = 99 \quad \sigma^2 = 7 \quad n = 25$$

②

جی رانیم  $E(\bar{X}) = \mu$  چون  $E\left(\frac{1}{n} \sum_{i=1}^n x_i\right) = \frac{1}{n} \sum_{i=1}^n \mu = \frac{n}{n} \mu = \mu$

$$V(\bar{X}) = \frac{\sigma^2}{n} \xrightarrow{\text{انتقال}} V\left(\frac{1}{n} \sum_{i=1}^n x_i\right) = \frac{1}{n^2} \sum_{i=1}^n V(x_i) = \frac{n \sigma^2}{n^2} = \frac{\sigma^2}{n}$$

سوں  $E(\bar{X}) = 99 \quad V(\bar{X}) = \frac{7}{25}$

$x_1, x_2, \dots, x \sim E(\theta) \rightarrow E(x) = \theta$

③

$$E(\bar{X}) = E\left(\frac{x_1 + x_2}{2}\right) = \frac{1}{2} E(x_1) + \frac{1}{2} E(x_2) = \frac{1}{2} \theta + \frac{1}{2} \theta = \theta$$

$$\sum_{i=1}^{51} y_i = 153 \quad \sum_{i=1}^{51} y_i^2 = 4945$$

④

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i = \frac{1}{51} \sum_{i=1}^{51} y_i = \frac{153}{51} = 3$$

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2 = \frac{1}{n-1} \left( \sum_{i=1}^n y_i^2 - n \bar{y}^2 \right)$$

$$\sum y_i^2 + n \bar{y}^2 - \frac{2 \bar{y} \sum y_i}{n} = \sum y_i^2 - n \bar{y}^2 = \frac{1}{50} (4945 - 51 \times 9) = \frac{205}{50} = 41$$

$T = a_1 x_1 + a_2 x_2 \quad E(T) = a_1 E(x_1) + a_2 E(x_2)$

⑤

$$= a_1 \mu + a_2 \mu$$

$$= (a_1 + a_2) \mu$$

سوں  $a_1 + a_2 = 1$

$$V(T) = a_1^2 V(x_1) + a_2^2 V(x_2) = \sigma^2 (a_1^2 + a_2^2) = \sigma^2 \sum_{i=1}^2 a_i^2 = \sigma^2 \left( \sum_{i=1}^n (a_i - \frac{1}{n})^2 + \frac{1}{n} + 2 \sum_{i=1}^n (a_i - \frac{1}{n}) \frac{1}{n} \right)$$

$$\textcircled{b} \Rightarrow a_i = \frac{1}{r} \quad T = \frac{1}{r} X_1 + \frac{1}{r} X_2 = \frac{X_1 + X_2}{r} = \bar{X}$$

$$T_2 = \frac{X_1 - 2X_2}{r} \quad V(T_2) = V\left(\frac{X_1 - 2X_2}{r}\right) = \frac{1}{r^2} V(X_1) + V(X_2)$$

$$\frac{1}{r^2} X_1 + \frac{1}{r^2} X_2 = \frac{1}{r^2} \sigma^2 + \sigma^2 = \frac{5}{r^2} \sigma^2$$

$$V(\bar{T}) = \frac{1}{r^2} V(X_1) + \frac{1}{r^2} V(X_2) = \frac{1}{r^2} \sigma^2 + \frac{1}{r^2} \sigma^2 = \frac{1}{r^2} \sigma^2$$

$$\Rightarrow \frac{V(T_2)}{V(\bar{T})} = \frac{\frac{5}{r^2} \sigma^2}{\frac{1}{r^2} \sigma^2} = \frac{5}{1} = 5$$

کتابت  $T$  و  $T_2$  برابر ۵ است.

$$\bar{X} = X_1 + X_2 + X_3 \quad \textcircled{c}$$

ثابت  $X_3 = X_3$   $X_1 \sim N(175, 25) \rightarrow \begin{cases} E(X_1) = 175 \\ V(X_1) = 25 \end{cases}$

$X_2 \sim N(75, 11) \rightarrow \begin{cases} E(X_2) = 75 \\ V(X_2) = 11 \end{cases}$

$$P(X > 312) = ? \quad E(X) = E(X_1 + X_2 + X_3)$$

$$= E(X_1) + E(X_2) + E(X_3)$$

$$= 175 + 75 + 50 = 300$$

$X \sim N \rightarrow aX + b \sim N$   
طبق خواص نزول  $X$  نزول است  
(جمع نزول ۴ نزول است)

چون مستقلند

$$V(X) = V(X_1) + V(X_2) + V(X_3)$$

$$= 25 + 11 + 0 = 36$$

$$P(X > 312) = P\left(\frac{X - \mu}{\sigma} > \frac{312 - 300}{6}\right) = P(Z > 2)$$

$$= 1 - P(Z < 2) = 1 - \Phi(2) = 1 - 0.9772 = 0.0228$$

$$\mu_1 = 0 \quad \sigma_1 = 3 \quad X_1 \text{ صاف}$$

$$\mu_2 = 9 \quad \sigma_2 = 4 \quad X_2 \text{ فوف}$$

$$X_1 \sim N(0, 3^2) \quad X_2 \sim N(9, 4^2)$$

$$Y = X_1 + X_2 \sim N(11, 25) \text{ دوسرے}$$

$$\text{وہی } P(2X_1 + 2X_2 < 18) = P(\underbrace{X_1 + X_2}_Y < 9)$$

$$= P\left(\frac{Y - \mu}{\sigma} < \frac{9 - 11}{5}\right) = P(Z < -0.4)$$

$$\% 17 \text{ سے } \sigma = 5 \times 0.1344 = 0.672 \text{ ہے}$$

$$= \Phi(-0.4) = 0.1544$$

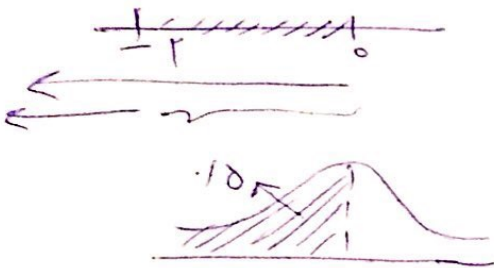
$$X \sim N(\mu, \sigma^2) \quad P(4\sigma < X < 6\sigma) = P\left(\frac{4\sigma - \mu}{\sigma} < Z < \frac{6\sigma - \mu}{\sigma}\right)$$

$$= P(-2 < Z < 0)$$

$$= P(Z < 0) - P(Z < -2)$$

$$= \Phi(0) - \Phi(-2)$$

$$= 0.5 - 0.054 = 0.446$$



$$X_1, \dots, X_n \quad X \sim \text{poisson}(\lambda) \rightarrow \mu = \lambda = E(X) \quad \textcircled{A}$$

$$\rightarrow V(X) = \lambda$$

$$E(\bar{X}) = \lambda \quad V(\bar{X}) = \frac{\lambda}{n} = \frac{1}{10}$$

$$P(\bar{X} > 1) = P\left(\frac{\bar{X} - \mu}{\sqrt{V(\bar{X})}} > \frac{1 - \lambda}{\sqrt{0.1}}\right) = P(Z > 4.123)$$

$$= 1 - P(Z < 4.123) = 1 - \Phi(4.123) = 0$$

